# Short Note 8.2 Further Discussion of Voronoi Diagrams

## Introduction

Chapter 8 presents evidence to support idea that undisclosed simulation model by Dr. Gottwald was the real research which lead the way to the 1900-ft rule. The model was capable of providing assessment of areal coverage and eradication estimates for various radii. Evidence in support of this conclusion are:

The eradication estimate would require a simulation program and eradication estimates were determined by Dr. Gottwald.

- Program that Dr. Gottwald used to calculate random quadrats would be essentially the same as the simulation program to calculate areal coverage and eradication estimates,
- Voronoi Tessellations as presented by Dr. Gottwald indicate determination of a radii than would result in high areal coverage. Areal coverage of 100% means circles result in all citrus being destroyed.

The viewgraph submitted into evidence, November 2000) is shown in Figure 1:





This viewgraph was presented by Dr. Gottwald in the Broward County District Court <sup>2</sup> and it is believed to be the same as presented at the International Citrus Canker Research Workshop in June 2000, because the footnote at the bottom states 2000 Citrus Canker Workshop June 2000.ppt.

The viewgraphs from the workshop have never been released, but a transcript of the talk was posted on the FDACS website. The discussion on the Voronoi diagram is as follows:

To start to analyze this data at the suggestion of Dr. Gareth Hughes, a friend of mine and colleague, we decided to try to look at the distribution from known focal trees to surrounding trees. To do that, we attempted to use what is known as Voronoi tessellation. This allowed us to carve up this data set such that if we used a focal tree and examined that, we knew anything that fell within this area (indicating) given to us by the tessellation was closer to this focal tree than any other focal tree in the area.

That worked very well for a few number of focal trees, but as the number of focal trees increased, that particular analytical technique no longer worked for us. This would be what it would look like if you overlaid it on top of, in this case, Site 1. We weren't able to then use this analysis, so we had to switch away from the Voronoi tessellation technique and developed our own program for estimating distances, and I'll talk about that, through a series of Visual Basic routines which analytically do the same thing as that technique which is a spatial analysis to try to determine how far citrus canker will spread from individual points of infection. <sup>2</sup>

The word "tessellations" means tiling. Voronoi tessellation refers to a method of subdividing a given area into subareas based on boundaries drawn equidistant from each point in the area.<sup>3</sup>

An example is shown below. In mining, Voronoi polygons are used to estimate the reserves of valuable materials, minerals, or other resources. Exploratory bore holes are used as the set of points in the Voronoi polygons.

#### **Figure 2: Example of Voronoi Tessellations**<sup>3</sup>



A defined set of boundaries is needed to enclose the polygons on the periphery. In the above example (Figure 2), the space has 22 points which define 22 corresponding polygons. Of these polygons, only 8

are entirely enclosed by Voronoi calculated sides and all others are bounded on at least one side by the outer boundaries of the space.

The selection of outer boundaries would likely have the greatest impact on the distribution of areal spaces when the number of points is small and clustered.

It is theorized that Voronoi tessellations were used perhaps early in the planning process to help define eradication radii with high areal coverages. The area of each bounded polygon can be calculated using commercial software programs. The areas could then be ranked in order, lowest to highest, and using percentiles, a large area equal to the 95% quantile could be identified. Then a radius equal to this large area would be calculated, to assure clear-cutting of 95% of all citrus trees in an area.

Alternatively, the distance between infected trees and the furthest corner of the polygon could be calculated. A similar analysis of these distances, using percentiles, could be used to identify a radius to clear-cut a high percentage of all citrus trees. This is illustrated in Figure 3.

#### Figure 3: Use of furthest corner to define eradication radius



The Voronoi calculations, whether based on enclosed areas or distance to the furthest vertex would likely result in high radii for new discoveries, which were distant away from known discoveries. This could be the result of new introductions of canker as a result of contamination in the nursery or commercial distribution areas.

### Applications

Voronoi diagram have been a very useful tool in many different areas including epidemiology. A list of some applications in areas of health, engineering, geometry and informatics is provided in the Wikipedia links. Wireless network coverage is one of the more recent applications as noted in the Wikipedia reference.

### Summary

Voronoi diagrams subdivide an area into polygons as discussed in reference 3. If the distribution of areas, or distance to the furthest vertex is the statistic of interest, then the results are likely affected by the outer boundaries.

In agricultural applications, the outer boundary would be the limit of the grove or orchard. However, for application of residential areas, where

Voronoi diagrams to identify areal coverages of eradication radii is speculative. However, I can think of no other reason except to calculate radii capable of high areal coverage.

### References:

1. Gottwald, T.R, 2000, Presentation to the Broward Court Case 00-18394 (08) CACE. (copies of viewgraphs provided on supporting documents website).

2. Gottwald, T.R., Sun, X., Riley, T., Graham, J.H., Hughes, G, 2000, Estimating Spread of Citrus Canker via Differential GPS, presentation made at the International Citrus Canker Research Workshop, transcript only.

3. Wikipedia, Voronoi Diagrams, https://en.wikipedia.org/wiki/Voronoi\_diagram